Name: ______________________ Date: _________________

Optimal Solutions

What is an Optimal Solution?
Definition: An **optimal solution** is a feasible **solution** (a region bounded by given constraints) where the objective function reaches its maximum or minimum value. For example, the most profit or the least cost.

Sample Problem
A store has requested a manufacturer to produce pants and sports jackets.
For materials, the manufacturer has **$750** of cotton textile and **$1,000** of polyester.
Every pair of pants needs **$1** of cotton and **$2** of polyester.
Every jacket needs **$1.5** of cotton and **$1** of polyester.
The price of the pants is **fixed** at **$50** and the jacket is **fixed** at **$40**.
What is the number of pants and jackets that the manufacturer must give to the stores so that these items obtain a **maximum** sale?

x = number of pants
y = number of jackets

Objective Function (maximum)
f (x,y) = 50x+40y

Constraints

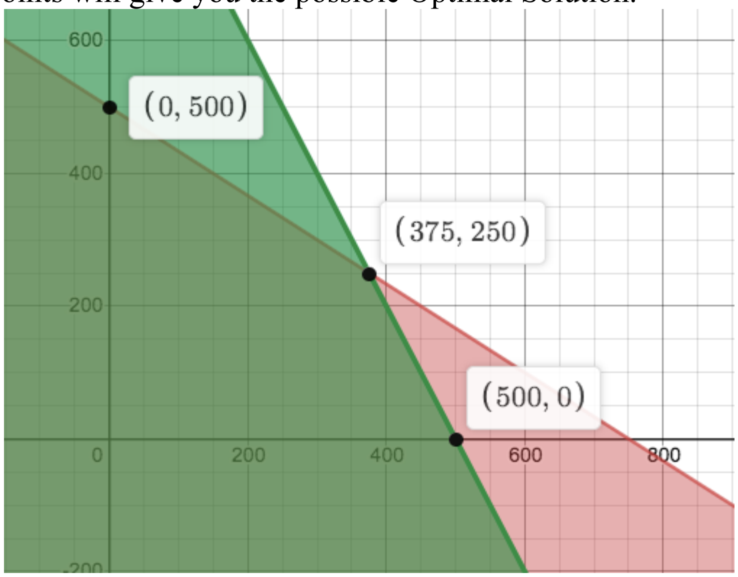
	pants	jackets	available
cotton	1	1,5	750
polyester	2	1	1,000

$x+1.5y \leq 750$
$2x+y \leq 1{,}000$
$x, y \geq 0$ (both x and y are non-negative)

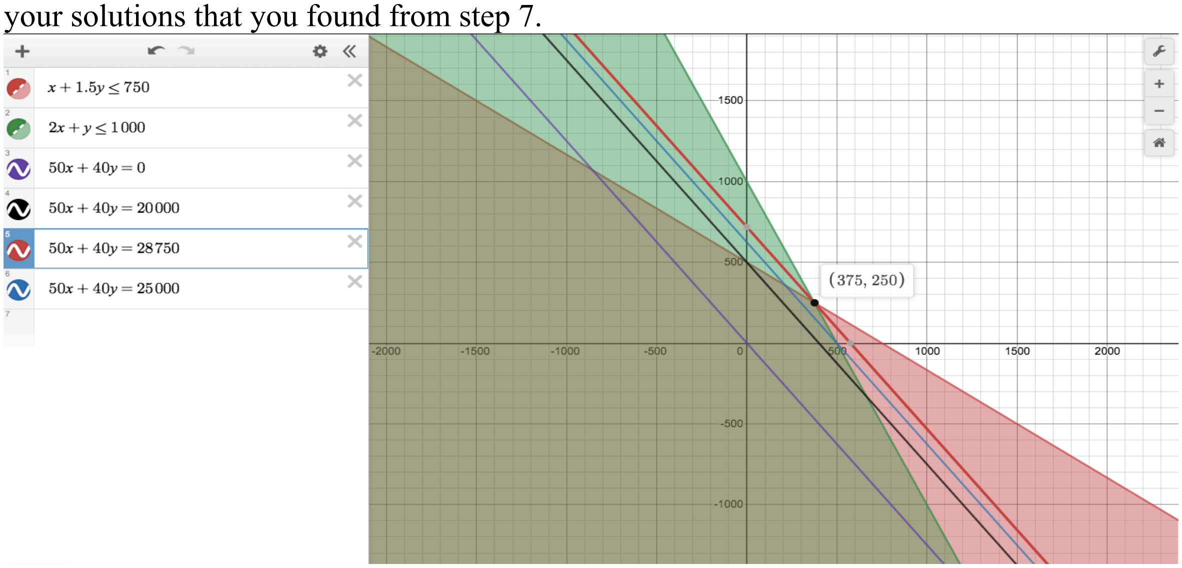
Steps to follow for Desmos

Step 1. Open up Desmos at https://www.desmos.com/calculator
Step 2. Click on the first line on the left for the keyboard to pop up and type in the first constraint ($x + 1.5y \leq 750$). You will see that Desmos automatically creates a line with proper shading.
Step 3. After you finish typing in your first constraint, you can either hit enter to get to the second line or click on the second line using your cursor. Put in the second constraint ($2x + y \leq 1{,}000$).

Step 4. After Desmos creates the line with the proper shading, on the right you will see a "+" and "-" sign. Click on those buttons to zoom in or out on your graph.
Step 5. Double tap on the extreme points. The extreme points are the points of intersection within the constraints which also includes the point (0,0). Put your cursor over each point and click twice. These points will give you the possible Optimal Solution.



Step 6. Using the extreme points you have found, calculate the maximum products into your calculators by using the objective function (f(x,y)=50x+40y).
You should get the following solutions:
50(0) + 40(0) = **0**
50(0) + 40(500) = **20,000**
50(375) + 40(250) = **28,750 *(Maximum)**
50(500) + 40(0) = **25,000**
Step 7. On lines 3 to 6, type in the objective function (50x+40y) and set the equation equal to your solutions that you found from step 7.



(The Optimal Solution is the point, (375, 250). This is the point where the max solution line hits last. This point is the maximum amount of product that can be made to reach the maximum profit.)

Practice Problems

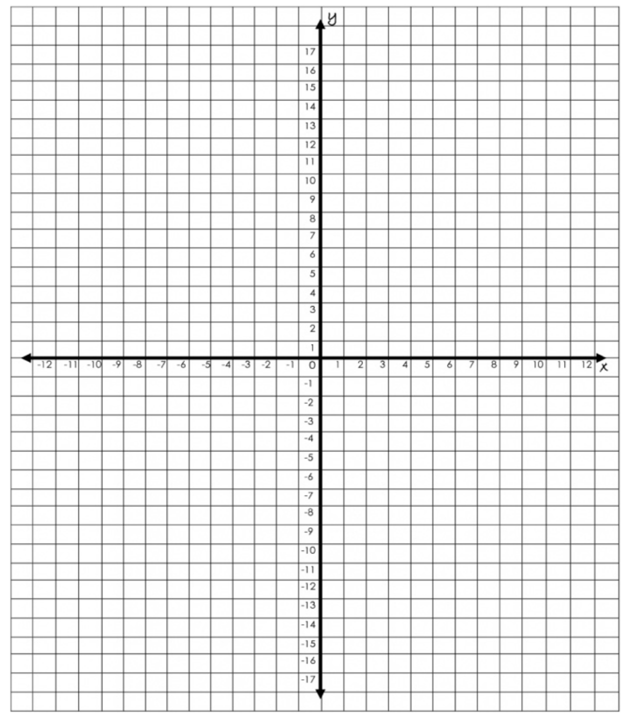
- **Use Desmos to find the Optimal Solution and Maximum/Minimum Value for both questions.**
- **Find the extreme points and write how many there are for both questions.**
- **Use the graphs to draw what is seen on Desmos.**

Question 1.

Objective Function
(Minimize)
F(x,y) = 2x+3y

Constraints
$3x + 6y \geq 24$
$3x + y \geq 9$
$x \geq 0$
$y \geq 0$

Number of Extreme Points: _____
Optimal Solution: _________
Minimum Value: _________



Question 2.

Objective Function
(Maximize)
F(x,y) = x+3y

Constraints
$x + y \leq 5$
$x + 2y \leq 8$
$x \geq 0$
$y \geq 0$

Number of Extreme Points: ______
Optimal Solution: _________
Maximum Value: _________